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# **Designing Robust Feedback Linearisation Controllers using Imperfect Dynamic Models and Sensor Feedback**

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# Designing Robust Feedback Linearisation Controllers using Imperfect Dynamic Models and Sensor Feedback

Abstract: The paper considers key limitation of the feedback linearisation controller designed for nonlinear systems based on the imperfect nominal dynamics model and sensors. The model-reality differences cause signal leakages in the feedback linearised dynamics. As the leakages are the functions of the process variables, the resulting overall dynamics are again nonlinear with strong additive nonlinearities and the expected decoupling of the system dynamics is missing. In the paper, instead of using advanced control tools, we prove the robustness of the feedback linearisation method can also be significantly enhanced by employing several simple and classical methods cooperatively. For clear description and explanation, the methodology was illustrated based on a two-link manipulator case study, a classical multi-input multi-output (MIMO) coupled nonlinear system. The methods have genetic potential so that they can be applicable to a variety of case study systems and also further developed to become general methodologies.

Keywords: Robotic manipulator, Robust design, Uncertainty, Signal leakages, Imperfect dynamic models

## 1. Introduction

Feedback linearisation control [1] has been widely used in controlling nonlinear and coupled multi-input multi-output (MIMO) systems [2]–[6]. Designing a feedback linearisation controller achieves the desired decoupling and linearisation only if the model is an accurate representation of reality. The model-reality differences cause signal leakages in the feedback linearised dynamics. As the leakages are the functions of process variables, the resulting overall dynamics are again nonlinear with strong additive nonlinearities. In addition, the expected decoupling of the system dynamics is missing and the resulting feedback linearisation controller is not robust enough to reach the desired performance. Such uncertainties are inevitable due to errors and wear in long-term usage.

Several tools have been developed to overcome uncertainties. Achieving robust decentralized architecture under strong link-reactions in robotic manipulators by applying the celebrated robust control technologies such as H-infinity is highly non-trivial. For the unlimited uncertainty, a number of adaptive control methods have been established for designing feedback linearisation controllers. A generic adaptive feedback linearisation controller with dynamic neural networks was developed for the control of uncertain nonlinear systems under no measurable states [7] and applied to high dynamic performance induction motor control [2]. The application of simple static neural networks to design adaptive controllers for manipulators under full measurement noise-free access to the needed variables was reported [8]. The generic adaptive feedback linearisation controller with Takagi-Sugeno fuzzy models adapted online was developed [9], [10]. The adaptive linearisation feedback controller was developed for two-link robot arm under varying load torque and exact mathematical model [11].

Global asymptotic stability of an adaptive output feedback tracking controller was developed for motion control of robot manipulator [12], [13].

However, robust controllers are not necessarily built on any fancy tools and concepts. They can also be achieved by combining simple, classical and well-established tools. In this paper, instead of presenting new tools, we present a general approach, or a methodology, to develop robust feedback linearisation controllers based on accessible basic and popular control methods widely used. Depending on the radius of the uncertainty, two approaches are employed in the paper to enhance the control performance: robust approach and adaptive approach. For clear description and explanation, the methodology was illustrated based on a two-link manipulator case study, a typical multi-input multi-output (MIMO) coupled nonlinear system, which can be controlled by using various tools including model predictive control [14] and slide mode control [15]–[19]. The approach described in this paper shows that robust control performance can also be achieved by using feedback linearisation even when big errors exist in nominal models and sensor feedback. We believe the strategy can be applied to a variety of systems and also further developed to become general methodologies.

The paper is organised as follows. Section 2 presents the mathematical model of a two-link robot arm and illustrates the feedback linearisation controller. Section 3 demonstrates a robust control approach for uncertain model parameters under a small uncertainty radius. When the uncertainty is significant due to large parameter errors and an existence of structural uncertainties in the nonlinear plant model dynamics, an adaptive approach is described in Section 4. The structural uncertainty means parts of the real model are missing in the nominal model.

## 2. Feedback linearisation control

### 2.1 Model of manipulator dynamic

The mechanical structure of a two-link robot arm, or two-link manipulator is shown in Figure 1. The mass  $m_1$  and  $m_2$  of each two links are uniformly distributed along their length  $L_1$  and  $L_2$ . The torque is applied to the two joints to drive the movement of each link. The dynamics model of a two-link robot arm shown as equation (1-4) is derived using the Lagrange method.

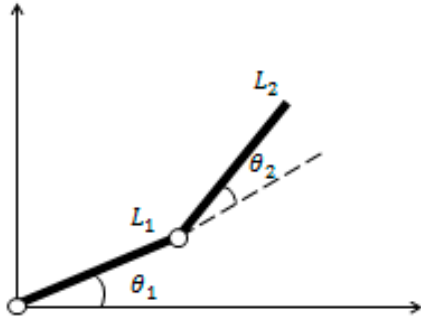


Figure 1 Two-link Manipulator Model

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) + F(\dot{\theta}) + \tau_d = \tau \quad (1)$$

$$M(\theta) = \begin{bmatrix} m_1 L_1^2 + m_2 L_2^2 + m_2 L_1^2 + 2m_2 L_1 L_2 \cos \theta_2 & m_2 L_2^2 + m_2 L_1 L_2 \cos \theta_2 \\ m_2 L_2^2 + m_2 L_1 L_2 \cos \theta_2 & m_2 L_2^2 \end{bmatrix} \quad (2)$$

$$C(\theta, \dot{\theta}) = \begin{bmatrix} -2\dot{\theta}_2 m_2 L_1 L_2 \sin \theta_2 & -\dot{\theta}_2 m_2 L_1 L_2 \sin \theta_2 \\ \dot{\theta}_1 m_2 L_1 L_2 \sin \theta_2 & 0 \end{bmatrix} \quad (3)$$

$$G(\theta) = \begin{bmatrix} m_1 L_1 g \cos \theta_1 + m_2 L_1 g \cos \theta_1 + m_2 L_2 g \cos(\theta_1 + \theta_2) \\ m_2 L_2 g \cos(\theta_1 + \theta_2) \end{bmatrix} \quad (4)$$

Where  $L_1, L_2$  - lengths,  $m_1, m_2$  - masses,  $g$ - acceleration of gravity,  $F(\dot{\theta})$ -Friction torques,  $\tau_d$ - disturbance torques,  $\tau_1, \tau_2$  - control torques. The torques  $\tau_1, \tau_2$  are the control inputs while the link positions  $\theta_1, \theta_2$  are the outputs of the system. This is a classical MIMO coupled nonlinear system.

## 2.2 Effects of uncertainties on control performance of feedback linearisation

When coupling the feedback linearisation mapping (FLM) to the plant, the acceleration  $a$  becomes the control inputs, producing the feedback linearised plant of the second order decoupled dynamics, as illustrated in Figure 2.

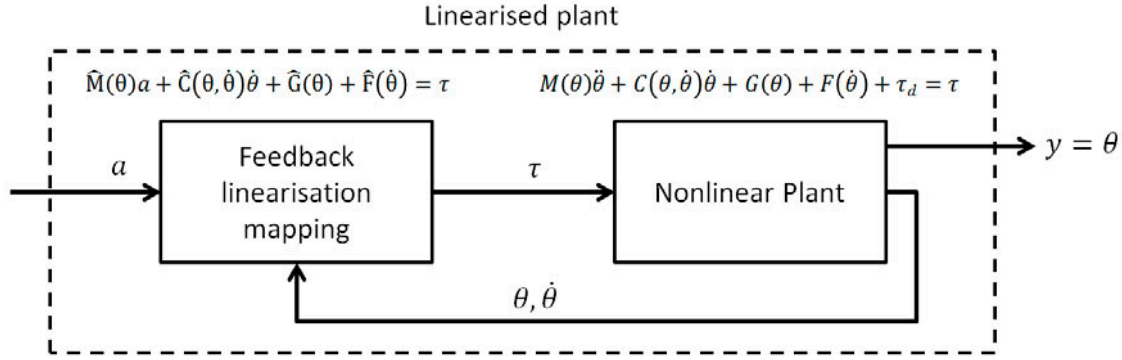


Figure 2 Linearised plant

The design of the FLM requires exact knowledge of the plant dynamics model (mappings  $M(\theta)$ ,  $C(\theta, \dot{\theta})$ ,  $G(\theta)$ ,  $F(\dot{\theta})$ ), disturbance inputs  $\tau_d$  and noise free plant variable values  $\theta(t)$ ,  $\dot{\theta}(t)$ . However, in reality only the approximate models  $\hat{M}(\theta)$ ,  $\hat{C}(\theta, \dot{\theta})$ ,  $\hat{G}(\theta)$ ,  $\hat{F}(\dot{\theta})$  of  $M(\theta)$ ,  $C(\theta, \dot{\theta})$ ,  $G(\theta)$ ,  $F(\dot{\theta})$  are available. These discrepancies will produce additive leakage signals (LS) in the feedback linearised plant dynamics, which can be nonlinear functions of the plant variables. The state space model of the manipulator becomes:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a - \text{LS} \end{aligned} \quad \text{where} \quad \begin{aligned} x_1 &= \theta \\ x_2 &= \dot{\theta} \end{aligned} \quad (5)$$

The leakage term can be derived accurately from the system equations, as shown in equation (6), when the uncertainty is only in the disturbance torques.

$$\text{LS}(t) = M^{-1}(\tau_d(t) - \tau(t)) \quad (6)$$

The linearised plant can be controlled by using PID controllers in the outer position loops. The control outcome of a two-link manipulator with  $L_1 = 20$ ,  $L_2 =$



15,  $m_1 = 0.7$ ,  $m_2 = 0.5$  is shown in Figure 3a, when the P, I, D terms equalled 10, 3, 0 respectively. However, good control performance is achievable only if the model is complete and parameters in the model are accurate. When the perfect model of the manipulator and noise free measurement are not available, the error between the real parameters and the estimated parameters introduces nonlinear and coupled leakages, as shown in Figure 3b when parameter errors are 50% (Real:  $L_1 = 20$ ,  $L_2 = 15$ ,  $m_1 = 0.7$ ,  $m_2 = 0.5$ ; Estimated:  $L_1 = 10$ ,  $L_2 = 7.5$ ,  $m_1 = 0.35$ ,  $m_2 = 0.25$ ).

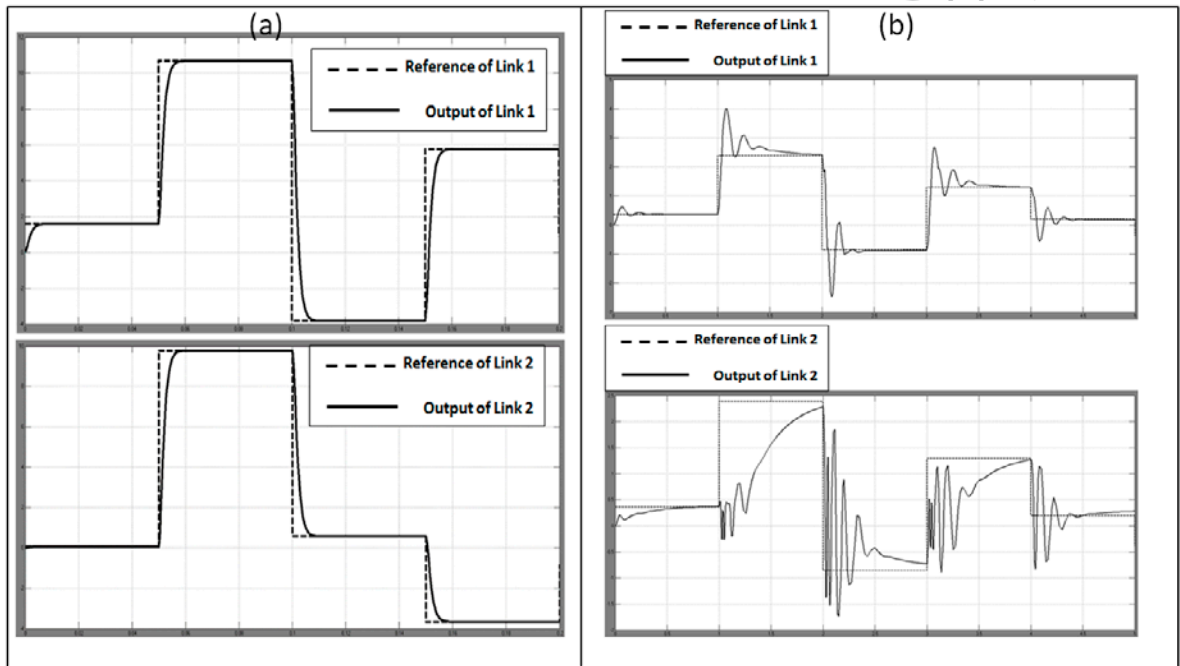


Figure 3 Effect of signal leakages in the feedback linearised dynamics. Control performance of (a) decentralized PID controller + Perfect Feedback linearisation mapping, and (b) decentralized PID controller + imperfect Feedback linearisation mapping.

The result indicates the impact of nonlinear and coupled leakages caused by systematic uncertainty on feedback linearisation controllers. In this study, we present two approaches employing simple control tools in a combined manner to form new configurations in the controllers to enhance the robustness of controllers based on feedback linearisation.

### 3. Robust control approach

Cascaded-loop controller design is a classic design to overcome nonlinear and coupled leakages through high gains applied in the inner velocity loop. Regarding the two-link manipulator case study, this design is able to provide satisfactory control performance when errors are in the radius of 100%, described in Section 3.1. However, the high gains increase system's sensitivity to measurement noise, resulting in large position tracking error. To keep the functionality of the cascaded-loop controller, we propose to use a Kalman filter to eliminate the noise so that the whole control system is able to robustly achieve required tracking accuracy, described in Section 3.2.

#### 3.1 Parameter uncertainties

Cascaded-loop design was used to overcome parameter uncertainties. Equation (6) shows that the LS caused by the parameter uncertainty appears entirely in the velocity loop. After implementing cascaded-loop configuration, the leakage introduced by the mechanical parameter uncertainty is compensated in the velocity loop or inner loop by using PI controller. While the outer loop provides a fast response and a high tracking accuracy assuming perfect performance of the inner velocity loop. In the inner loop, the P, I term in the two decentralized controllers are 30, 13 and 35, 20 respectively. The outer loop for each link is designed to employ a derivative feed-forward controller and a proportional controller with gain  $K_p$ .

The design of the outer loop controller starts from equation (7):

$$\int (\dot{\theta}^{\text{ref}} + (\theta^{\text{ref}} - \theta)K_p) = \theta \quad (7)$$

Differentiating equation (7) and rearranging the obtained terms yields the position error dynamics mode as follows:

$$\dot{\theta}^{\text{ref}} + (\theta^{\text{ref}} - \theta)K_p = \dot{\theta} \quad (8)$$

$$\dot{E} = \dot{\theta}^{\text{ref}} - \dot{\theta} = -K_p E \quad (9)$$

$$\therefore E = \exp(-K_p \cdot t) \quad (10)$$

From equation (10), by using a proportional controller and derivative feed forward controller in the outer loop, the position reference tracking error converges to zero regardless of the initial error. The convergence rate can be arbitrarily fast by suitable choice of the gain  $K_p$ .

### 3.2 Noise

For feedback linearisation mapping, the velocity and position measurement is used to compute desired torques. Noisy measurement introduces a large error to the computation results and control performance, especially when high gains are used in the inner loop.

A Kalman filter can be used to eliminate the noise. In our study, the acceleration of each link was measured by using acceleration sensors to predict the positions and velocities. The positions were also directly measured by using position sensors. According to the regular form of the continuous-time Kalman filter, the following equations are derived.

$$\dot{x} = Ax + Bu + m \quad (11)$$

$$y = Cx + n \quad (12)$$

$$p(m) \sim N(0, Q_c) \quad (13)$$

$$p(n) \sim N(0, R_c) \quad (14)$$

The state vector and corresponding matrices can be set in the following form:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} position \\ velocity \end{bmatrix} \quad (15)$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad 0] \quad (16)$$

Assume the noisy measurement of positions follows Gaussian distribution and the power of the reference signal is 1 and variance of noise m and n can be as large as 0.1.

$$Q_c = \begin{bmatrix} 0 \\ \sqrt{0.1} \end{bmatrix} [0 \quad \sqrt{0.1}] = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix} \quad (17)$$

$$R = 0.1 \quad (18)$$

The estimated result is

$$\hat{x} = A\hat{x} + Bu + K(y - C\hat{x}) \quad (19)$$

The Kalman filter gains are updated as Eq. (21) and (22):

$$\dot{P} = -PC^TR_c^{-1}CP + AP + PA^T + Q_c \quad (20)$$

$$K = PC^TR_c^{-1} \quad (21)$$

When the system operates under a piecewise constant disturbance of high frequency and a noise with SNE = 1 dB and the parameter errors are 50%, the tracking results are shown in Figure 4. The newly designed controller achieves an average tracking error=0.5%, overshoot= 0% and settling time= 0.01s. This design minimised the effects of noise due to suitable high gains in cascaded-loop configurations, and increase the robustness of the system.

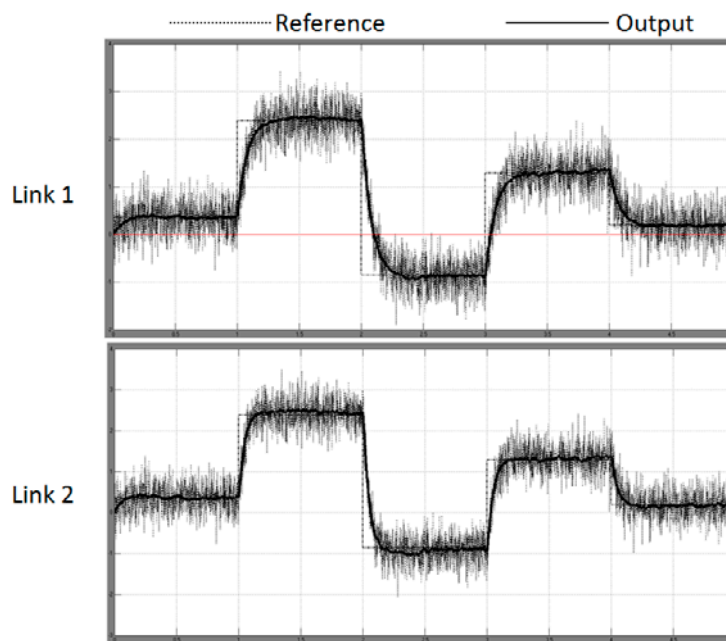


Figure 4 (a) Control performance of two separated PID controller + Imperfect Feedback Linearisation mapping + Double cascaded loop controller for piecewise constant input signal and piecewise constant disturbance with noisy measurement; (b) Control performance of the designed robust controller

Overall, the robustness of feedback linearisation controller can be improved by using classical cascaded-loop configuration without amplifying the effects of sensor noise due to high gains. The design could provide satisfactory control performance when parameter errors are in the radius of 100%.

#### 4. Adaptive control approach

The robust design can ensure a good control performance when the parameter errors stay in the radius of 100%. However, when uncertainty exists in a larger radius, such as large parameter errors over 100% and incomplete dynamic models (or structural uncertainty), the previous robust control approach is not strong enough. In terms of two-link manipulator dynamics, the stiction friction is missing in the original dynamic models (1-4) and introduces structural uncertainty. In this section, we present a simple adaptive controller adopting a number of simple and basic control tools and algorithms

to correct parameters online (Section 4.1) and overcome unknown structural uncertainty (Section 4.2) simultaneously.

#### 4.1. Large parameter errors

A sub-system called a parameter estimator is added to the system in order to update the current parameter values based on the comparison of the real and predicted/estimated outputs. The parameter update algorithm aims at forcing the output mismatch to zero.

Figure 5 describes the new configuration of the system.

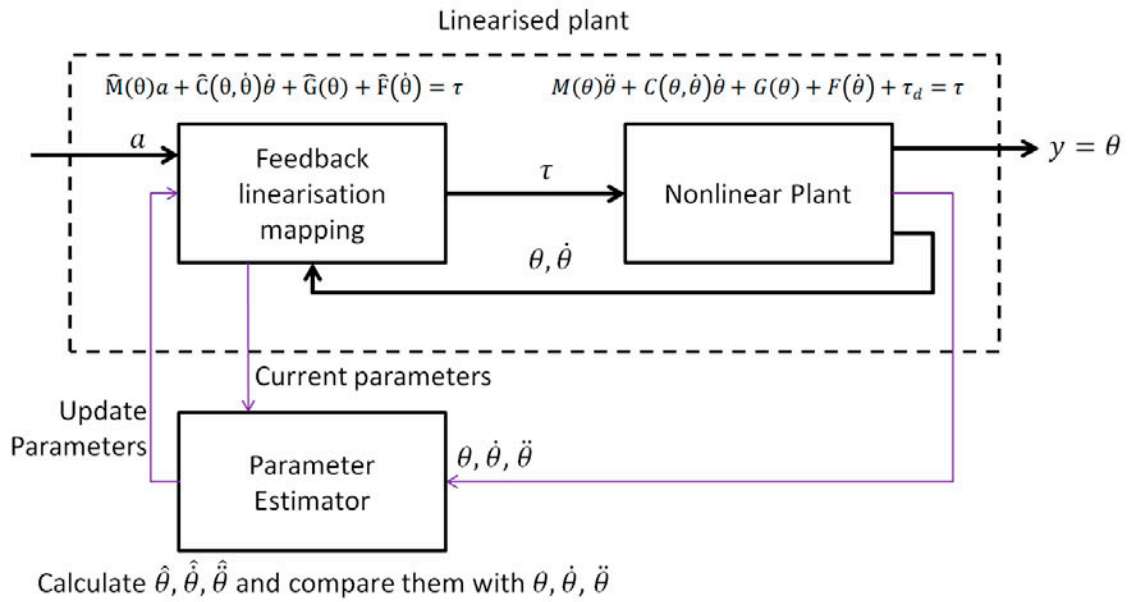


Figure 5 Block diagram of the control system containing a parameter estimator

Considering the relationship between the plant and the feedback linearisation mapping, the following equations are derived from equation (1-5).

$$M^{-1}(\theta)[\hat{M}(\theta)a + \hat{C}(\theta, \dot{\theta})\dot{\theta} + \hat{G}(\theta) + \hat{F}(\dot{\theta})] - F(\dot{\theta}) - G(\theta) - C(\theta, \dot{\theta})\dot{\theta} = \ddot{\theta} \quad (22)$$

$$(\hat{M} - M)\ddot{\theta} + \hat{M}(a - \ddot{\theta}) + (\hat{C} - C)\dot{\theta} + (\hat{G} - G) = 0 \quad (23)$$

$$\Delta M \ddot{\theta} + \hat{M} e + \Delta C \dot{\theta} + \Delta G = 0 \quad (24)$$

In (24),  $e$  is the error between the measured acceleration and the expected acceleration into the feedback linearisation mapping while  $\Delta$  denotes the parameter errors on the values of the real and nominal model mappings. The aim of the parameter estimator is to solve the equation  $\Delta M \ddot{\theta} + \hat{M} \dot{e} + \Delta C \dot{\theta} + \Delta G = 0$  and generate new parameter values to update the old ones.

In equation (24)

$$\Delta M = \hat{M} - M$$

$$= \begin{bmatrix} \Delta(m_1 L_1^2) + \Delta(m_2 L_2^2) + \Delta(m_2 L_1^2) + 2\Delta(m_2 L_1 L_2) \cos \theta_2 & \Delta(m_2 L_2^2) + \Delta(m_2 L_1 L_2) \cos \theta_2 \\ \Delta(m_2 L_2^2) + \Delta(m_2 L_1 L_2) \cos \theta_2 & \Delta(m_2 L_2^2) \end{bmatrix} \quad (25)$$

$$\Delta C = \hat{C} - C = \begin{bmatrix} -2\Delta(m_2 L_1 L_2) \dot{\theta}_2 \sin \theta_2 & -\Delta(m_2 L_1 L_2) \dot{\theta}_2 \sin \theta_2 \\ \Delta(m_2 L_1 L_2) \dot{\theta}_1 \sin \theta_2 & 0 \end{bmatrix} \quad (26)$$

$$\Delta G = \hat{G} - G = \begin{bmatrix} \Delta(m_1 L_1) g \cos \theta_1 + \Delta(m_2 L_1) g \cos \theta_1 + \Delta(m_2 L_2) g \cos(\theta_1 + \theta_2) \\ \Delta(m_2 L_2) g \cos(\theta_1 + \theta_2) \end{bmatrix} \quad (27)$$

Assuming that

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{bmatrix} = \begin{bmatrix} \Delta(m_1 L_1^2) \\ \Delta(m_2 L_1^2) \\ \Delta(m_2 L_2^2) \\ \Delta(m_2 L_1 L_2) \\ \Delta(m_1 L_1) \\ \Delta(m_2 L_1) \\ \Delta(m_2 L_2) \end{bmatrix} \quad (28)$$

The equations (25-27) can be transformed to (29-31)

$$\Delta M = \begin{bmatrix} X_1 + X_2 + X_3 + 2X_4 \cos \theta_2 & X_3 + X_4 \cos \theta_2 \\ X_3 + X_4 \cos \theta_2 & X_3 \end{bmatrix} \quad (29)$$

$$\Delta C = \begin{bmatrix} -2X_4 \dot{\theta}_2 \sin \theta_2 & -X_4 \dot{\theta}_2 \sin \theta_2 \\ X_4 \dot{\theta}_1 \sin \theta_2 & 0 \end{bmatrix} \quad (30)$$

$$\Delta G = \begin{bmatrix} X_5 g \cos \theta_1 + X_6 g \cos \theta_1 + X_7 g \cos(\theta_1 + \theta_2) \\ X_7 g \cos(\theta_1 + \theta_2) \end{bmatrix} \quad (31)$$

Thus, the whole function (24) can now be transformed to (32)

$$K(\ddot{\theta}, \dot{\theta}, \theta)X + \hat{M}(\hat{m}_1, \hat{m}_2, \hat{L}_1, \hat{L}_2)e = 0 \quad (32)$$

Where

$$K = \begin{bmatrix} \ddot{\theta}_1 & \ddot{\theta}_1 & \ddot{\theta}_1 + \ddot{\theta}_2 & (2\ddot{\theta}_1 + \ddot{\theta}_2) \cos \theta_2 - 2\dot{\theta}_1 \dot{\theta}_2 \sin \theta_2 - \dot{\theta}_2^2 \sin \theta_2 & g \cos \theta_1 & g \cos \theta_1 & g \cos(\theta_1 + \theta_2) \\ 0 & 0 & \ddot{\theta}_1 + \ddot{\theta}_2 & \ddot{\theta}_1 \cos \theta_2 + \dot{\theta}_1^2 \sin \theta_2 & 0 & 0 & g \cos(\theta_1 + \theta_2) \end{bmatrix} \quad (33)$$

$\hat{M}(\theta)$

$$= \begin{bmatrix} \hat{m}_1 \hat{L}_1^2 + \hat{m}_2 \hat{L}_2^2 + \hat{m}_2 \hat{L}_1^2 + 2\hat{m}_2 \hat{L}_1 \hat{L}_2 \cos \theta_2 & \hat{m}_2 \hat{L}_2^2 + \hat{m}_2 \hat{L}_1 \hat{L}_2 \cos \theta_2 \\ \hat{m}_2 \hat{L}_2^2 + \hat{m}_2 \hat{L}_1 \hat{L}_2 \cos \theta_2 & \hat{m}_2 \hat{L}_2^2 \end{bmatrix} \quad (34)$$

$$e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} a_1 - \ddot{\theta}_1 \\ a_2 - \ddot{\theta}_2 \end{bmatrix} \quad (35)$$

equation (33) is valid at any time instant. In order to calculate parameters  $L_1$ ,  $L_2$ ,  $m_1$  and  $m_2$ , the data is sampled in a time stream at a different time instant so that  $K$  is expanded to  $K_t$  and thus equation (32) is transformed into equation (36).

$$X_t = -K_t^{-1}(\ddot{\theta}, \dot{\theta}, \theta) \hat{M}_t(\hat{m}_1, \hat{m}_2, \hat{L}_1, \hat{L}_2) e_t \quad (36)$$

When sampling three times,

$$X_t = \begin{bmatrix} X_{t1} \\ X_{t2} \\ X_{t3} \end{bmatrix} = - \begin{bmatrix} K_{t1} \\ K_{t2} \\ K_{t3} \end{bmatrix}^{-1} \begin{bmatrix} \hat{M}_{t1} \\ \hat{M}_{t2} \\ \hat{M}_{t3} \end{bmatrix} \quad (37)$$

There are three different time instants,  $t_1, t_2, t_3$ , that  $K$ ,  $X$  and  $M$  sampled at. As



seen in (33),  $K$  at different time  $t$  is always following a certain proportional law. Thus the maximum rank of  $K_t$  is 5 and there is infinite number of roots for  $X$ .  $K_t$  can be simplified and transformed into Equation (38)

$$K_t = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (38)$$

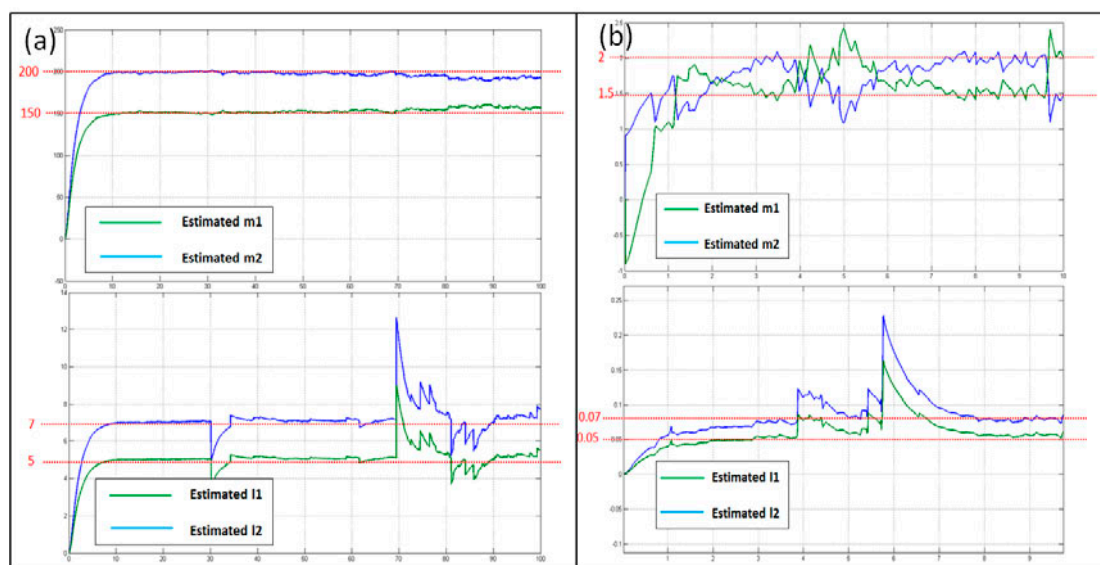
The pseudo-inverse operator is a tool to solve this problem. The estimated results of  $X_3$ ,  $X_4$  and  $X_7$  are accurate. Therefore, assuming that the mass of the whole manipulator is available, the value of parameters can be calculated online by using the equations below.

$$\left. \begin{aligned} \hat{m}_2 \hat{L}_2^2 - X_3 &= m_2 L_2^2 = Y_1 \\ \hat{m}_2 \hat{L}_1 \hat{L}_2 - X_4 &= m_2 L_1 L_2 = Y_2 \\ \hat{m}_2 \hat{L}_2 - X_7 &= m_2 L_2 = Y_3 \\ \frac{Y_1}{Y_3} &= L_2 \\ \frac{Y_2}{Y_3} &= L_1 \\ \frac{Y_3^2}{Y_1} &= m_2 \\ mass - m_2 &= m_1 \end{aligned} \right\} \quad (39)$$

To test the algorithm, it is assumed that the initial parameters in the feedback linearisation mapping are fixed and two parameter data sets are used in the real dynamic models: extremely large values and extremely small values, as shown in Table 1. The performance of the parameter estimator using the algorithm described above is given in Figure 6.

Table 1 Ultra-large error

parameter error: 1000%	Feedback linearization mapping				Real dynamic model			
	$m_1$	$m_2$	$L_1$	$L_2$	$m_1$	$m_2$	$L_1$	$L_2$
<b>Large parameters</b>	20	15	0.7	0.5	200	150	7	5
<b>Small parameters</b>	20	15	0.7	0.5	2	1.5	0.07	0.05

Figure 6 Estimated values of  $m_1$ ,  $m_2$ ,  $L_1$  and  $L_2$ . (a) Ultra-large parameters; (b) Ultra-small Parameters.

When the real parameters are 10 times larger or smaller than the parameters in the feedback linearisation mapping, the estimated parameter fluctuates approximately within a 10% range around the target value. The parameter-learning algorithm is effective since it reduces the parameter error from 1000% to around 10%, which are in the working radius of the robust controller.

#### 4.2 Structural uncertainties

According to the stiction phenomena, when the robot arm starts to move and the velocity is small, it experiences high torque at its joints. Thus, stiction friction depends on the velocity in a nonlinear way. However, this part is missing in the nominal model, causing structural uncertainties. The relationship between the stiction friction and velocity in the steady-state is shown in Figure 7.

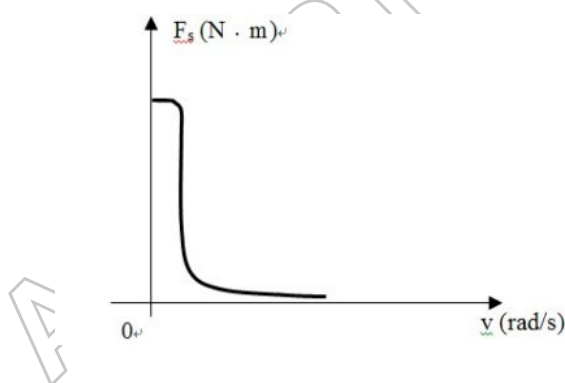


Figure 7 Continuously changing approximation of relationship between static torque and velocity

The stiction friction torque can be expressed by an exponential function shown as follows:

$$F_s(v) = f_s \cdot \exp\left(-\frac{v^2}{v_s}\right) \quad (40)$$

In (40),  $f_s$  is the maximum value of the stiction friction and  $v_s$  is a parameter set to be a very small value. Stiction frictions  $F_{s1}$  of link 1 and  $F_{s2}$  of link 2 are indicated by (41) and (42) respectively.

$$F_{s1}(\dot{\theta}_1) = f_s \cdot \exp\left(-\frac{\dot{\theta}_1^2}{v_s}\right) \cdot \text{sign}(\dot{\theta}_1) \quad (41)$$

$$F_{s2}(\dot{\theta}_2) = f_s \cdot \exp\left(-\frac{\dot{\theta}_2^2}{v_s}\right) \cdot \text{sign}(\dot{\theta}_2) \quad (42)$$

The overall friction matrix  $F$  in (1) is expanded to Equation (43):

$$F(\dot{\theta}) = \begin{pmatrix} f_{11}\dot{\theta}_1 + f_{12}\text{sign}(\dot{\theta}_1) + f_s \cdot \exp\left(-\frac{\dot{\theta}_1^2}{v_s}\right) \cdot \text{sign}(\dot{\theta}_1) \\ f_{21}\dot{\theta}_2 + f_{22}\text{sign}(\dot{\theta}_2) + f_s \cdot \exp\left(-\frac{\dot{\theta}_2^2}{v_s}\right) \cdot \text{sign}(\dot{\theta}_2) \end{pmatrix} \quad (43)$$

The stiction torque expressed by (43) constitutes a real model of the friction. The leakage signal in the dynamical model (6) of the feedback linearised system equals to:

$$LS = M^{-1}F_s(\dot{\theta}) \quad (44)$$

Where inertia matrix  $M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$  with none of its components zero,

and  $F_s = \begin{bmatrix} F_{s1} \\ F_{s2} \end{bmatrix}$ , where  $F_{s1}$  and  $F_{s2}$  are defined in (41) and (42) respectively. Hence,

$$M^{-1}F_s(\dot{\theta}) = \frac{1}{|M|} \begin{pmatrix} m_{22}F_{s1}(\dot{\theta}_1) - m_{12}F_{s2}(\dot{\theta}_2) \\ -m_{21}F_{s1}(\dot{\theta}_1) + m_{11}F_{s2}(\dot{\theta}_2) \end{pmatrix} \quad (45)$$

Therefore, for each link, the leakage into the link due to the uncertainty in the stiction friction varies with velocities of both links in a nonlinear manner. Simulation results (Figure 8) of the system with integral controllers under the stiction friction illustrate the impact of static friction on the tracking performance.

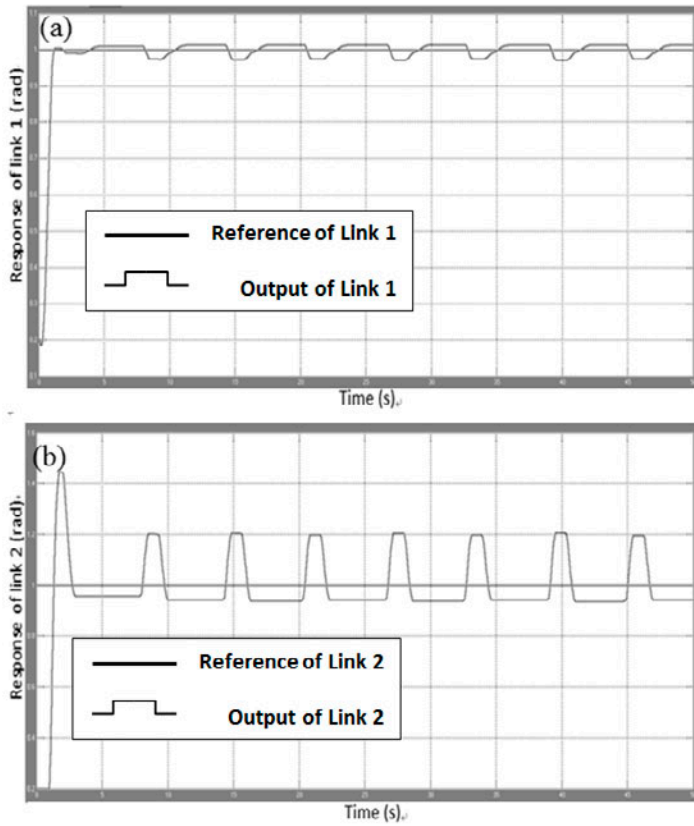


Figure 8 Impact of stiction friction that missed in the nominal model

This structural uncertainty can be compensated simply by using a classical gain-scheduling mechanism. The linguistic setting details of the gain scheduling are illustrated in Figure 9.

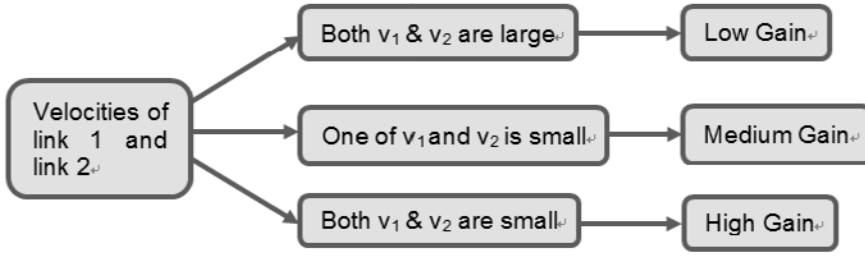


Figure 9 Linguistic setting details of gain scheduling

As the stiction leakages directly appear in the inner loops, the gain scheduling is only applied to the PI controllers in the inner loops. This helps to stop/reduce the impacts transferred to the outer position loops. Taking into account that the actuator is unable to generate rapidly changing torques, soft gain scheduling is used, where changes of the gain values are distributed over time as opposed to the hard switching of the gains.

The soft gain scheduling is shown in (46).

$$GS_i(\dot{\theta}) = \alpha_{i,1} \exp(-\dot{\theta}_1^2 / v_s) + \alpha_{i,2} \exp(-\dot{\theta}_2^2 / v_s) + 1 \quad (46)$$

where  $i=1,2$  denotes the link number.

The control performance is given in Figure 10, by using the parameter estimator to correct parameters and the gain scheduling to compensate structural uncertainties (stiction friction). Due to the inner loop including a gain-scheduled PI controller, the control system is able to overcome not only static torque, but also piecewise-constant disturbance due to the integral fast controller. When the parameter error is as large as 1000% and structural uncertainty exists in the form of stiction friction, It is demonstrated that the steady state error of link 1 lower than 0.01% and the steady state error of link 2 lower than 0.03%.

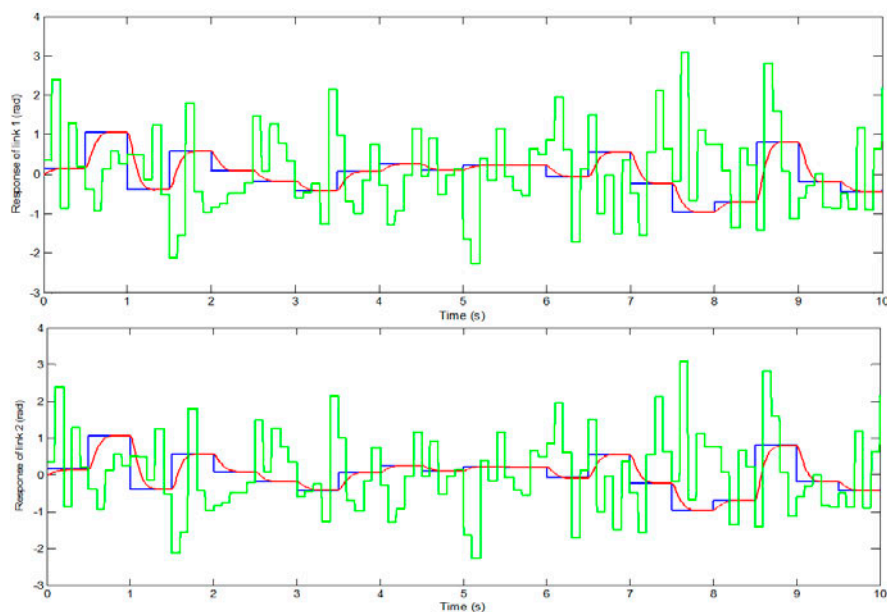


Figure 10 Control performance of the designed adaptive controller. Actual outputs (red); Reference outputs(blue); Disturbance(green)

Overall, the robustness of feedback linearisation controller can be improved by using a simple parameter learning algorithm and a classical gain-scheduling mechanism. When parameter errors are 1000%, this new and simple approach is capable of achieving a good tracking performance.

## 5. Conclusion

This paper proves that instead of using a unique advanced tool, the robustness of feedback linearisation controller can be significantly improved by a combination of simple and basic tools. This general approach, or methodology, is able to overcome significant uncertainties in nominal models and sensors. The methodology was presented based on a two-link manipulator case study. When uncertainty stays in a small radius (parameter error < 100%), it is proposed to employ a robust control approach, in which cascaded-loop design and Kalman filter are used cooperatively, as the former is

able to overcome parameter errors through high gains in the inner loop, while the latter prevents amplifying the noise owing to such high gains. When uncertainty is in a large radius, it is proposed to use an adaptive control approach consisting of a simple parameter learning algorithm and gain-scheduling mechanism to correct parameters and eliminate structural uncertainties. As different tools dealing with different aspects of uncertainties, the technique can be further supplemented and tailored to fit requirements, and become a general methodology.

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## Public Interest Statement

In many applications, it is possible to use linear equations to describe the behaviour of the system accurately enough and linear controllers could work well enough if the system operates near the equilibrium. However, if it deviates too far from the equilibrium, the linear equations are no longer valid and linear controllers will fail. Feedback linearisation is a common approach used in controlling nonlinear systems. However, the control performance heavily depends on the accuracy of nominal models and sensors, which are possible to vary due to errors and wears in a long term usage. Such uncertainties set a limit of the implementation of feedback linearisation design. Here, we present an approach to improve the robustness of a feedback linearisation controller by collaborating simple and classical tools to overcome a wide range of uncertainties in nominal models and sensors.